Exercise 13

Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$$p(v) = 2\sqrt{3v^2 + 1}, \quad a = 1$$

Solution

By definition, a function is continuous at a number a if

$$\lim_{v \to a} p(v) = p(a).$$

Evaluate the function at $v = 1$.

$$p(1) = 2\sqrt{3(1)^2 + 1} = 2\sqrt{4} = 4$$

Calculate the limit as v approaches 1 using the limit laws.

$$\begin{aligned}
\lim_{v \to 1} p(v) &= \lim_{v \to 1} 2\sqrt{3v^2 + 1} \\
&= 2 \lim_{v \to 1} \sqrt{3v^2 + 1} \\
&= 2\sqrt{\lim_{v \to 1} (3v^2 + 1)} \\
&= 2\sqrt{\lim_{v \to 1} 3v^2 + \lim_{v \to 1} 1} \\
&= 2\sqrt{3 \lim_{v \to 1} v^2 + 1} \\
&= 2\sqrt{3\left(\lim_{v \to 1} v\right)\left(\lim_{v \to 1} v\right) + 1} \\
&= 2\sqrt{3(1)(1) + 1} \\
&= 4
\end{aligned}$$

The condition in the definition is satisfied, so $p(v) = 2\sqrt{3v^2 + 1}$ is a continuous function at $a = 1$.